

Exotic Multi-quark States in the Deconfined Phase from Gravity Dual Models

P. Burikham*, A. Chatrabhuti†, E. Hirunsirisawat‡

*Theoretical High-Energy Physics and Cosmology Group, Department of Physics,
Faculty of Science, Chulalongkorn University, Bangkok 10330, Thailand*

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Abstract

In the deconfined phase of quark-gluon plasma, it seems that most of the quarks, anti-quarks and gluons should be effectively free in the absence of the linear confining potential. However, the remaining Coulomb-type potential between quarks in the plasma could still be sufficiently strong that certain bound states, notably of heavy quarks such as J/ψ are stable even in the deconfined plasma up to a certain temperature. Baryons can also exist in the deconfined phase provided that the density is sufficiently large. We study three kinds of exotic multi-quark bound states in the deconfined phase of quark-gluon plasma from gravity dual models in addition to the normal baryon. They are k -baryon, $(N + \bar{k})$ -baryon and a bound state of j mesons which we call “ j -mesonance”. Binding energies and screening lengths of these exotic states are studied and are found to have similar properties to those of mesons and baryons at the leading order. Phase diagram for the exotic nuclear phases is subsequently studied in the Sakai-Sugimoto model. Even though the exotics are less stable than normal baryons, in the region of high chemical potential and low temperature, they are more stable thermodynamically than the vacuum and chiral-symmetric quark-gluon plasma phases (χ S-QGP).

*Email:piyabut@gmail.com

†Email:auttakit.c@chula.ac.th

‡Email:ekapong.hirun@gmail.com

1 Introduction

The discovery of AdS/CFT correspondence [1, 2] provides a new tool for studying the strongly coupled gauge theories. Although the original setup and most of the systems that string theorists have been investigating so far are highly supersymmetric and conformal, a lot of progress has been made in constructing more realistic models. Now we have examples of QCD-like gauge theory with known gravity dual that share most of the qualitative features of QCD. These holographic models allow us to perform analytic calculations in the regimes which are too difficult to implement for the real QCD even for lattice calculations. The properties of quark-gluon plasma from Relativistic Heavy Ion Collisions and QCD at finite baryon density are two examples of such regimes.

The gravity dual of baryons can be described via baryon vertex [3, 4], a D-brane wrapping higher dimensional sphere in 10-dimensional curved background with N strings attached to it and ending at the boundary. These strings are required to cancel an N charge in the world-volume of the wrapped brane due to the presence of RR flux in the background. The endpoint of fundamental string that ends on D-brane is electrically charged with respect to world-volume $U(1)$ gauge field. Its charge is $+1$ or -1 depending on the orientation of the string and D-brane. Moreover, strings stretching from the baryon vertex to the boundary of AdS or the corresponding background spacetime (e.g. in Sakai-Sugimoto model) behave as fermions, giving antisymmetry to the baryon vertex. This fact allows us to construct an $SU(N)$ gauge-invariant combination of N quarks as required by the group theory. Baryon configurations were investigated further in [5]-[7]. The authors in [8] extended the consideration in confining background where it was found that the binding energy is linear in N and in the size of the baryon on the boundary. And furthermore, they found that in $\mathcal{N}_{SUSY} = 4$ theory there are stable configurations for baryons which are made of k quarks, or “ k -baryon”, if $5N/8 < k \leq N$. Such configurations can be realized by considering the usual baryon vertex with k strings stretched up to the boundary and the rest $N - k$ strings stretched down to the horizon. These baryons are not colour singlet and transform as $\frac{N!}{k!(N-k)!}$ representation under $SU(N)$ gauge group, for example the case $k = N - 1$ gives rise to a baryonic configuration in the anti-fundamental representation. In a confining theory we do not expect to find such a bound state. It was proposed in [9, 10] that the $k < N$ bound states can exist in a deconfined phase.

In general, we could imagine that there would be more exotic baryon states in the deconfined phase where bound states of quarks need not be the colour singlet. Some attempts have been made in constructing holographic description of exotic multi-quarks bound states[9]-[12]. The author in [12] considered exotic quark configurations formed by combining two or more baryon vertices together. However, it might be possible to construct an exotic baryon from a single baryon vertex which should be more energetically preferable. One useful observation is that there are infinite combinations of string charges that can cancel the charge from the background RR flux. Hence, the total number of strings attached to the baryon vertex need not to be equal to N . For example, if the orientation of D-branes is fixed in such a way that there is $+N$ units of charge on its world-volume, we can attach $N + k$ strings, each with -1 charge and k strings with $+1$ charge to make the total charge vanishes. As

long as the conservation of charge is concerned, k could be any integer. In this case, we can construct a $k > N$ baryon. Such baryon could be the lightest bound state in some irreducible representation of the underlying gauge theory thus it may be stable and can be observed in the deconfined phase. We would like to investigate this possibility further in this paper.

It is also interesting to study exotic baryons in more realistic model such as Sakai-Sugimoto model [13, 14]. This model is based on Witten's model [15] which uses the D4-brane wrapping a Scherk-Schwarz circle and adds a stack of N_f probe D8-branes and a stack of N_f probe anti-D8-branes transverse to the circle. This model contains massless chiral fermions and the flavour symmetry. The most striking feature of this model is that it introduces geometrical mechanism for spontaneous chiral symmetry breaking. Using the fact that the circle vanishes at a finite radial coordinates in the near horizon limit, D8-branes and anti-D8-branes are connected in a U-shaped configuration. At low temperature the model describes a confining gauge theory with broken chiral symmetry. Above a deconfinement temperature, gluons become effectively free. However, both the connected U-shape D8-branes configuration and the separated parallel brane-anti-brane configuration are possible in the intermediate temperature. The chiral symmetry is still broken even though the gluons are already deconfined. At higher temperature the chiral symmetry is restored, which is illustrated geometrically by the separation of the D8-branes and anti-D8-branes [16]. This corresponds to the branes being in parallel configuration.

The model also has an interesting phase structure. Finite baryon density in the Sakai-Sugimoto model has been studied in [17, 18] and extended to the full parameter space in [19] where baryon matter is represented by D4-branes in the D8-brane (nuclear matters) or by strings stretched from the D8-brane down to the horizon (quark matters). It was shown that the former configuration is always preferred to the latter and quark matters are unstable to density fluctuations. In the deconfined phase there are three regions corresponding to the vacuum, quark-gluon plasma, and nuclear matter, with a first-order and a second-order phase transition separating these three phases. The author in [19] found that for a large baryon number density, and at low temperatures, the dominant phase has broken chiral symmetry in agreement with QCD at high density. It is interesting to see how exotic baryon states fit into the phase structure.

This paper is organized as the following. In section 2, we discuss some classes of exotic baryon configurations and investigate their static configurations in section 3. Binding energy and screening length of the configurations are calculated in section 4. The dependence on free quark mass of exotic baryon configuration is discussed in section 5. The phase diagram of Sakai-Sugimoto model with exotic baryons is investigated in section 6. We discuss our results in section 7 and conclude in section 8.

2 Some classes of multi-quark states

In the deconfined phase of QGP, coloured states of a number of quarks and antiquarks can exist in the medium as long as it is energetically more favoured than the free quarks and antiquarks or other mesonic states. We will call these multi-quark states as “baryons” in

this article. In the confined phase, the only allowed baryons are those with colour singlet combinations such as nucleons and pentaquarks. For the deconfined phase, baryons can have colour and thus can have more varieties than the situation in the confined phase.

In general, a $D(8-p)$ -brane wrapping the subspace S_{8-p} of the background spacetime sources the gauge field $A_{(1)}$ on its world volume. This gauge field will couple with the antisymmetric $(8-p)$ -form field strength $G_{(8-p)}$ and induce the charge upon the wrapping $D(8-p)$ -brane. If the background is generated by a stack of N Dp -branes, then the charge being induced upon the wrapping $D(8-p)$ -brane will be exactly N . This charge needs to be cancelled by external charges brought about by strings. Each of the strings stretching out from the wrapping brane to the spacetime boundary or probe branes carries -1 unit of charge. Therefore it is required that the total number of “quark” strings stretching out from the wrapping brane must be N . The configuration of wrapping $D(8-p)$ -brane with totally N strings stretching out is called a baryon vertex [3, 4].

For the confined phase, since quarks cannot exist as free-quark strings with one end falling behind the horizon, therefore they can only start from the baryon vertex and go to the probe branes. On the other hand, in the deconfined phase, a radial string configuration lying along the radial coordinate is also a classical solution of the Nambu-Goto action [20] and it represents the free (anti)quark state in the QGP medium. A string can either start from the baryon vertex and go radially to the horizon of the background spacetime or it can come from the horizon and end at the baryon vertex. We will call this string configuration which is allowed in the deconfined phase as the “radial string”.

In the deconfined phase of QGP, it is possible to have k_h strings hanging from the space-time boundary down to the baryon vertex and another k_r strings stretching radially from the baryon vertex down to the horizon. The total number $k_h + k_r = N$ is the charge conservation constraint on the configuration. This configuration is known as “ **k -baryon**” [8].

Another possible configuration is when there are N quark-strings and \bar{k} antiquark-strings hanging down to the vertex from the probe branes. To conserve the charge, there are additional k quark-strings hanging from the vertex down to the horizon. We will call this configuration “ **$(N + \bar{k})$ -baryon**” (e.g. pentaquark could be represented by one of this kind).

An even more interesting configuration allowed in the deconfined phase is when there are j pairs of quark and antiquark strings hanging from the probe branes down to the vertex. Again, to conserve charges, we need N radial strings stretching from the vertex down to the horizon. This configuration obviously can decay into j mesons when it is less energetically favoured. Therefore we will call this state, a “ **j -mesonance**”, representing a binding state of j mesons in the QGP.

In summary, the charge conservation constraint for each case can be expressed as the following.

For k -baryon,

$$k_h + k_r = N; \quad k_h = k. \quad (1)$$

For $(N + \bar{k})$ -baryon,

$$k_h - k_r = N; \quad k_h = N + \bar{k}. \quad (2)$$

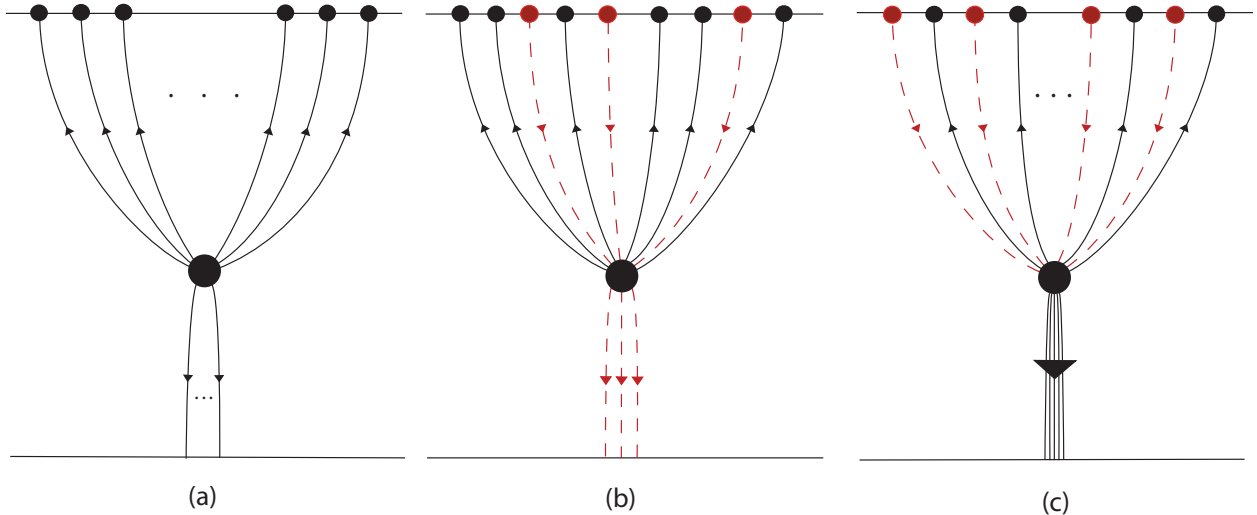


Figure 1: The gravity dual configurations of the hypothetical exotic states (a) k -baryon with the number of hanging strings $k_h = k < N$ and the number of radial strings $k_r = N - k$. (b) $(N + \bar{k})$ -baryon with $k_h = N + \bar{k}$ and $k_r = \bar{k}$. (c) j -mesonance with $k_h = 2j$ and $k_r = N$.

For j -mesonance,

$$k_h = 2j; \quad k_r = N. \quad (3)$$

Note that k_h is the number of strings hanging from the boundary down to the baryon vertex and k_r is the number of strings hanging from the vertex down to the horizon. The value of \bar{k} and j can be as large as $N \times N_f$. However, in this article, we will take this number to be large and ignore the upper bound on \bar{k} and j . Each configuration of exotic baryons is illustrated in Fig. 1.

3 Force conditions

In this section, we will consider the force condition for each exotic configuration of the quarks and antiquarks in a deconfined phase. The calculation will be performed in the gravity background similar to those of Sakai and Sugimoto's [13]. Even though the chiral symmetry restoration can be addressed within this model, we will not consider the aspect in this section but rather focus our attention on the high temperature phase where quarks and antiquarks are effectively free in the absence of the linear confining potential. The positions of $D8/\overline{D8}$ will be taken to be large and we will approximate it to be infinity in this section as well as in the discussion of binding energy and screening length in section 4. Analysis in this heavy-quark limit provides us with valuable physical understanding of certain essential features of the exotic states. Generalized results for a near-horizon background metric of the Dp -branes solution and its dependence on positions of the probe branes will be given in section 5.

Even in the deconfined phase, quarks and antiquarks feel effective (screened) potential

from other constituents. Therefore, a number of population of them will exist in various forms of bound states, some of which are exotic in the sense that they cannot be formed in the confined phase at low temperature.

Start with the following background metric

$$ds^2 = \left(\frac{u}{R_{D4}}\right)^{3/2} (f(u)dt^2 + \delta_{ij}dx^i dx^j + dx_4^2) + \left(\frac{R_{D4}}{u}\right)^{3/2} \left(u^2 d\Omega_4^2 + \frac{du^2}{f(u)}\right)$$

$$F_{(4)} = \frac{2\pi N}{V_4} \epsilon_4, \quad e^\phi = g_s \left(\frac{u}{R_{D4}}\right)^{3/4}, \quad R_{D4}^3 \equiv \pi g_s N l_s^3,$$

where $f(u) \equiv 1 - u_T^3/u^3$, $u_T = 16\pi^2 R_{D4}^3 T^2/9$. Note that the compactified x_4 coordinate (x^4 transverse to the probe D8 branes), with arbitrary periodicity $2\pi R$, never shrinks to zero. The volume of the unit four-sphere Ω_4 is denoted by V_4 and the corresponding volume 4-form by ϵ_4 . $F_{(4)}$ is the 4-form field strength, l_s is the string length and g_s is the string coupling. The dilaton in this background has u -dependence and its value changes along the radial direction u . This is a crucial difference in comparison to the AdS-Schwarzschild metric case where dilaton contribution is constant.

The action of the baryon configuration is given by

$$S = S_{D4} + k_h S_{F1} + k_r \tilde{S}_{F1}, \quad (4)$$

where S_{D4} represents the action of the D4-brane. S_{F1} is the action of a stretched string from the boundary down to the baryon vertex and \tilde{S}_{F1} is the action of a radial string hanging from the baryon vertex down to the horizon. Recall that S_{D4} can be obtained from the Dirac-Born-Infeld action¹. After some calculations, we obtain

$$S_{D4} = \frac{\tau N u_c \sqrt{f(u_c)}}{6\pi\alpha'}, \quad S_{F1} = \frac{\tau}{2\pi\alpha'} \int_0^L d\sigma \sqrt{u'^2 + f(u) \left(\frac{u}{R}\right)^3}, \quad \tilde{S}_{F1} = \frac{\tau}{2\pi\alpha'} (u_c - u_T), \quad (5)$$

where τ is the total time over which we evaluate the action and u_c is the position where the D4-brane vertex is located.

The variation of the action with respect to u gives the volume term and the surface term. The volume term leads to the usual Euler-Lagrange equation for the classical configuration of strings. As an approximation, we assume the baryon vertex to be a point (not being distorted by the connecting strings) located at a fixed value of $u = u_c$ as in Ref. [8]. Under this assumption, the surface terms provide additional *zero-force condition* on the configuration,

$$\frac{N}{3} G_0(x) - k_h B + k_r = 0 \quad (6)$$

¹

$$S_{DBI} = \int dx^0 d\xi^p T_p; \quad T_p = \left(e^{-\phi} (2\pi)^p \alpha'^{(p+1)/2}\right)^{-1} \sqrt{-\det(g)}$$

where

$$G_0(x) \equiv \frac{1 + \frac{x^3}{2}}{\sqrt{1 - x^3}}, \quad x \equiv \frac{u_T}{u_c} < 1, \quad \text{and} \quad B \equiv \frac{u'_c}{\sqrt{u_c'^2 + f(u_c)(\frac{u_c}{R_{D4}})^3}}. \quad (7)$$

Notice that these conditions occur at the location of the vertex at $u = u_c$, at which there exists the balance between the pull-up force (toward the direction of increasing u) due to the tension of hanging strings and the pull-down force due to the “weight”² of D4-brane plus the tension of radial strings.

Since $B \leq 1$, we obtain

$$k_h \geq \frac{N}{3} G_0(x) + k_r, \quad (8)$$

which expresses the lower bound of the number of hanging strings. In other words, the number of hanging strings cannot be less than this critical value, otherwise the no-force condition is not satisfied. The equality of (8) is held only when all hanging strings are stretched straight, otherwise we require more hanging strings to balance the pull-down force. Let us now consider each class of the multi-quark states.

In the case of **k -baryon**, plugging the condition (1) into (8), we obtain

$$k_h = k \geq \frac{N}{6} (G_0(x) + 3). \quad (9)$$

Apart from the lower bound, we also have the upper bound, $k \leq N$, therefore $G_0(x)$ cannot be larger than 3, resulting in

$$x \lesssim 0.922. \quad (10)$$

Notice that this restriction on x is a result from the conditions of the force balance and conservation of string charges. This shows that there is an upper-bound on the temperature, over which the horizon is too near to the point vertex that the pull-down force always overcomes the pull-up one.

In the case of **$(N + \bar{k})$ -baryon**, in the same way as the preceding case, plugging the condition of charge conservation (2) into (8), we have the following condition,

$$k_h = N + \bar{k} \geq \frac{N}{3} G_0(x) + \bar{k}.$$

Unlike the case of k -baryon, the upper-bound of the number of hanging strings does not exist. However, we still obtain the same condition $G_0(x) \leq 3$, hence $x \lesssim 0.922$.

Finally, in the case of **j -mesonance**, similarly, Eqn. (3) results in

$$j \geq \frac{N}{6} (G_0(x) + 3). \quad (11)$$

²This is not exactly the *weight* in the usual sense since the direct gravitational force on Dbrane is already balanced by the force from the RR-flux, but it is the force originated from minimization of self-energy due to the brane tension caused by the background metric and the gauge interaction. This is very similar to the self-energy of a spring under gravity where the spring potential energy changes with the tidal force from gravity in the background. The DBI action of the $D4 \sim u_c \sqrt{f(u_c)}$ which is positive for $u_c > u_T$ and becomes zero (minimum) at $u_c = u_T$ and thus it represents the “weight” on D4 towards the horizon.

The lower-bound of the value of j is $2N/3$ at zero temperature ($x = 0$) and it will be larger as the temperature grows. Nevertheless, the upper-bound of the limit on j does not exist.

Finally, we would like to comment on the limits on the value of k, \bar{k}, j when the temperature is zero. In terms of $n \equiv 7 - p$ (of the spacetime background generated by D p -branes), the condition (8) becomes

$$k_h \geq \frac{N}{n} + k_r \quad (12)$$

which leads to

$$\frac{k}{N}, \frac{j}{N} \geq \frac{n+1}{2n}, \quad (13)$$

and no conditions on \bar{k} . This critical numbers are $5/8, 2/3$ for $n = 4, 3$ (the AdS-Schwarzschild and Sakai-Sugimoto model) respectively. It is an interesting coincidence that the critical numbers are the same for both k -baryon and j -mesonance. Even though it appears from Eqn. (12) that there should also be a constraint on the $(N + \bar{k})$ configuration, it turns out that there is none.

4 Binding energy and the screening length

In this section we will calculate the binding energies of the k -baryon, $(N + \bar{k})$ -baryon, and j -mesonance in the deconfined phase. These binding energies are taken to be the differences between the total energies of each configuration and the corresponding energies of the free strings configuration which represents the free quarks and/or antiquarks state. The number of free strings in the free quarks state is determined solely by the total number of strings hanging from the boundary, k_h .

The total energy of each configuration is given by $E = S/\tau$ of the corresponding action S for each configuration. The binding energy for each hanging string is consequently,

$$E_{F1} = \frac{1}{2\pi} \int_0^L d\sigma \sqrt{u'^2 + \left(\frac{u}{R_{D4}}\right)^3 f(u)} - \frac{1}{2\pi} \int_{u_T}^\infty du. \quad (14)$$

Due to the no-force condition in the surface term, we impose Eqn. (6) and Eqn. (7), or

$$u_c'^2 = \frac{f(u_c)B^2}{1 - B^2} \left(\frac{u_c}{R_{D4}}\right)^3 \quad (15)$$

where the tension of each hanging string at u_c is constrained by

$$B = B(k_h, k_r, x) = \frac{N}{3k_h} G_0(x) + \frac{k_r}{k_h}. \quad (16)$$

Since the Lagrangian \mathcal{L} does not depend on σ explicitly, the conserved Hamiltonian can be defined to be

$$\mathcal{H} \equiv \mathcal{L} - u' \frac{\partial \mathcal{L}}{\partial u'} = \text{const}, \quad (17)$$

leading to

$$\frac{f(u_c)(\frac{u_c}{R_{D4}})^3}{\sqrt{u_c'^2 + f(u_c)(\frac{u_c}{R_{D4}})^3}} = \frac{f(u)(\frac{u}{R_{D4}})^3}{\sqrt{u'^2 + f(u)(\frac{u}{R_{D4}})^3}}. \quad (18)$$

Then substituting Eqn. (15) into this equation, we obtain

$$u'^2 = \frac{f(u)^2(\frac{u}{R_{D4}})^6}{f(u_c)(\frac{u_c}{R_{D4}})^3(1-B^2)} - f(u) \left(\frac{u}{R_{D4}} \right)^3. \quad (19)$$

This gives the size (radius) of the baryon as seen on the gauge theory side,

$$L = \frac{R_{D4}^{3/2}}{u_c^{1/2}} \int_1^\infty dy \sqrt{\frac{(1-x^3)(1-B^2)}{(y^3-x^3)(y^3-x^3-(1-x^3)(1-B^2))}}. \quad (20)$$

Note that $u_c \approx \frac{R_{D4}^3}{L^2}$ at the leading order.

Using Eqn. (19) and let $y \equiv u/u_c$, the regulated binding energy now becomes

$$E_{F1} = \frac{u_c}{2\pi} \left\{ \int_1^\infty dy \left[\sqrt{\frac{y^3-x^3}{(y^3-x^3)-(1-x^3)(1-B^2)}} - 1 \right] - (1-x) \right\}. \quad (21)$$

Hence, we obtain the total energy of the configurations as

$$E = \frac{Nu_T}{2\pi} \left(\frac{\sqrt{1-x^3}}{3x} + \left(\frac{k_h}{N} \right) \frac{\mathcal{E}}{x} + \left(\frac{k_r}{N} \right) \frac{1-x}{x} \right) \quad (22)$$

$$\sim \frac{N^2}{L^2} \quad (23)$$

where \mathcal{E} represents the terms within the brace of (21).

To obtain the relations between the total energy of the configurations $E(x)$ and $L(x)$, we eliminate the parameter $x = u_T/u_c$. By numerical calculations, the results are shown in Fig. 2,3. The binding energy of N -baryon is the deepest, suggesting that it is the most tightly bound state. For $(N+\bar{k})$ -baryon, increasing \bar{k} makes the binding energy smaller and the bound state is less tightly bound. The case of j -mesonance is quite similar. Generically, a j -mesonance has shallower binding potential than the total energy of j mesons. However, as j grows, the difference gets smaller and smaller.

The screening radius or screening length of exotic multi-quark state is defined to be the value of radius L^* at which the binding energy becomes zero from negative values at smaller distances. This screening radius is therefore one-half of the usual definition of screening length in the discussion of mesonic state where it is defined as the zero-potential distance between quark and antiquark.

Numerical results suggest that the screening length of baryons and mesonance decrease as the temperature increases, i.e. $L^* \sim 1/T$ for a fixed value of k, \bar{k}, j as is shown in Fig. 4-6. This is the generic form for the screening length in both the AdS-Schwarzschild and Sakai-Sugimoto models because it is the quantity which does not depend on the 't Hooft coupling

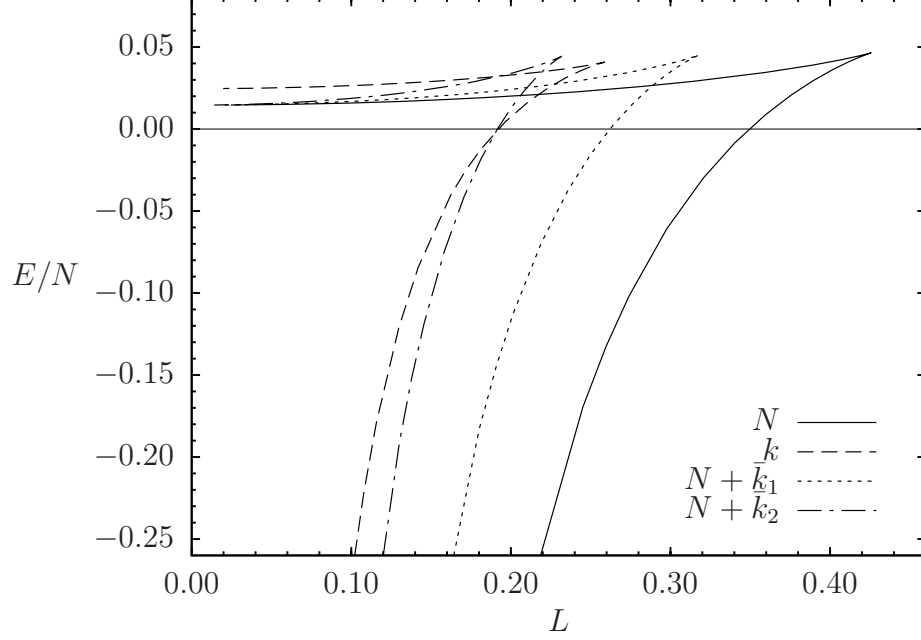


Figure 2: Comparison of the potential per N between N -baryon, k -baryon, and $(N + \bar{k})$ -baryon for $k/N = 0.8, \bar{k}_1/N = 2/3, \bar{k}_2/N = 2$ at temperature $T = 0.25$.

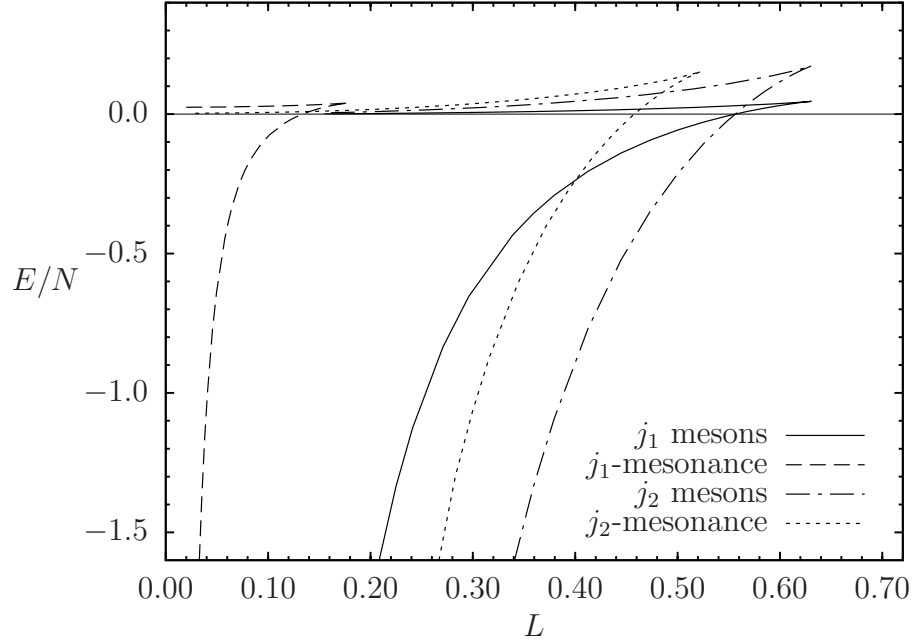


Figure 3: Comparison of the potential per N between j -mesonance and j mesons for $j_1/N = 0.8, j_2/N = 3$ at temperature $T = 0.25$.

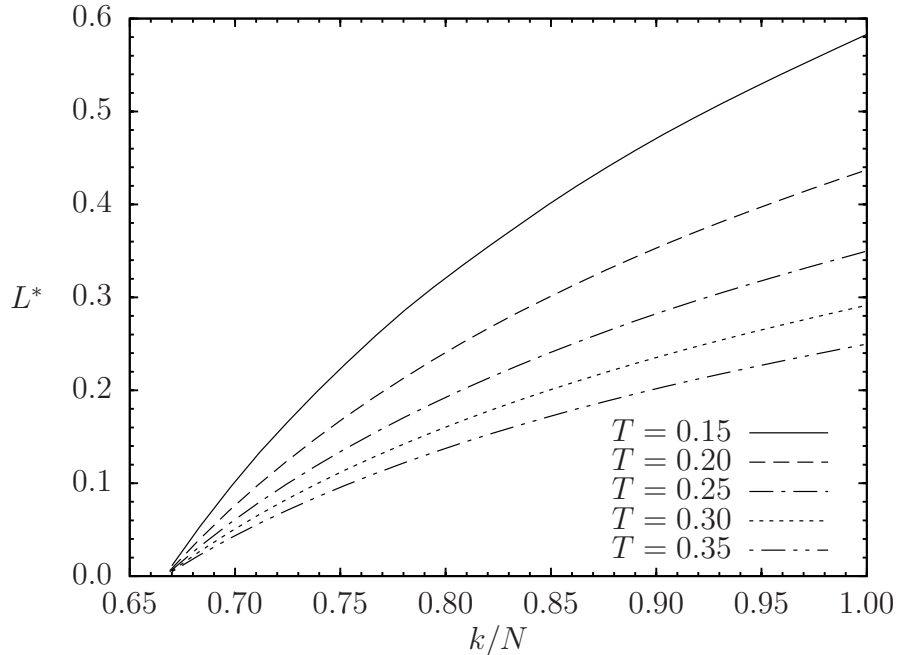


Figure 4: Screening length with respect to k for the temperatures in $0.15 - 0.35$ range.

at the leading order [21]. It is also an increasing function of k and j . Interestingly, $(N + \bar{k})$ -baryon has the opposite tendency with the screening length decreases as \bar{k} grows. On the other hand, the screening length of j -mesonance has a saturation value $L_{j\text{-mesonance}}^* \rightarrow L_{meson}^*$ as $j \rightarrow \infty$.

5 Dependence on the free quark mass

In this section, we will study dependence of the binding potential on the position of the probe branes. This is useful when position of the probe branes are at finite distance from the black hole horizon and the corresponding quarks have finite mass. For example, the probe branes are D8 and $\overline{\text{D8}}$ flavour branes in the Sakai-Sugimoto model.

The calculation of binding energy as a function of the radius L of the multi-quark states in the previous sections can be generalized to the case where the background metric is generated by a stack of Dp -branes as the following. Start with the energy of a hanging fundamental string with $n = 7 - p$,

$$E_{F1} = \frac{u_c}{2\pi} \left\{ \int_1^\infty dy \left[\sqrt{\frac{y^n - x^n}{(y^n - x^n) - (1 - x^n)(1 - A(n)^2)}} - 1 \right] - (1 - x) \right\} \quad (24)$$

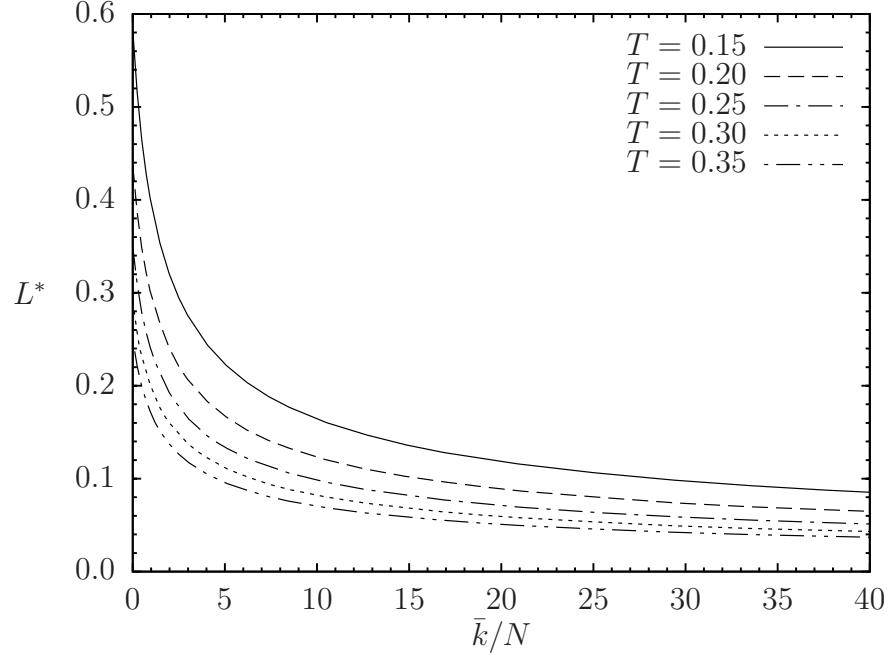


Figure 5: Screening length with respect to \bar{k} for the temperatures in 0.15 – 0.35 range.

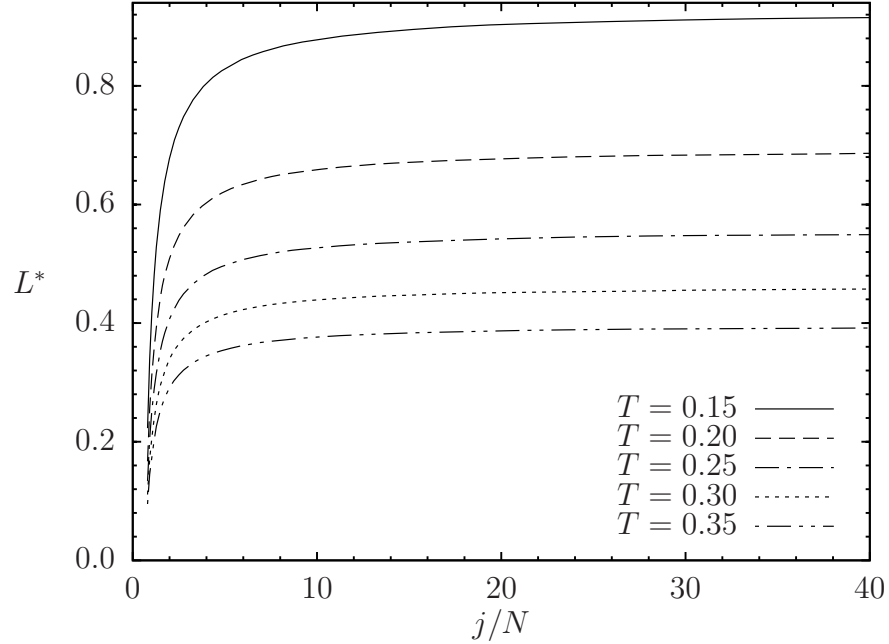


Figure 6: Screening length with respect to j for the temperatures in 0.15 – 0.35 range.

and the radius,

$$L = \frac{R^{n/2}}{u_c^{(n-2)/2}} \int_1^\infty dy \sqrt{\frac{(1-x^n)(1-A(n)^2)}{(y^n-x^n)(y^n-x^n-(1-x^n)(1-A(n)^2))}}. \quad (25)$$

The total regulated binding energy of the configuration then becomes

$$E_{tot} = \frac{Nu_h}{2\pi} \left\{ \frac{\sqrt{1-x^n}}{nx} + \left(\frac{k_h}{N}\right) \frac{\mathcal{E}}{x} + \left(\frac{k_r}{N}\right) \frac{1-x}{x} \right\} \quad (26)$$

where

$$\mathcal{E} = \int_1^\infty dy \left[\sqrt{\frac{y^n-x^n}{(y^n-x^n)-(1-x^n)(1-A(n)^2)}} - 1 \right] - (1-x), \quad (27)$$

and

$$A(n) = \frac{u'_c}{\sqrt{u_c'^2 + f(u_c)(\frac{u_c}{R_{Dp}})^n}} = \frac{N}{nk_h} \left(\frac{1 + \frac{n-2}{2}x^n}{\sqrt{1-x^n}} \right) + \frac{k_r}{k_h}. \quad (28)$$

The parameter x is again given by

$$x = \frac{u_T(n)}{u_c}, \quad u_T(n=3,4) = \frac{16}{9}\pi^2 R^3 T^2, \pi R^2 T. \quad (29)$$

Note that the case $n=3$ and $n=4$ corresponds to the case of Sakai-Sugimoto and AdS-Schwarzschild gravity dual model respectively.

Introduction of quark masses into the configuration can be done by terminating hanging strings at certain radial distance $u_{max} < \infty$. The universal behaviour of heavy-quark potential comes from the limit $u_{max} \rightarrow \infty$. We can split the total binding potential of the string into two parts. The first part is the binding potential in the $u_{max} \rightarrow \infty$ limit and the second part is the mass dependent potential. Therefore, the mass dependence part of the binding potential, $E_{F1}(u_{max})$ ($m = u_{max}/2\pi$), can be expressed as

$$E_{F1}(\text{finite mass}) = E_{F1}(u_{max} \rightarrow \infty) + E_{F1}(u_{max}), \quad (30)$$

$$E_{F1}(u_{max}) = -\frac{u_c}{2\pi} \int_{u_{max}/u_c}^\infty dy \left[\sqrt{\frac{y^n-x^n}{(y^n-x^n)-(1-x^n)(1-A(n)^2)}} - 1 \right] \quad (31)$$

$$= -\frac{u_{max}(1-A(n)^2)}{4\pi(n-1)} \left(\frac{u_c^n - u_T^n}{u_{max}^n} \right) + O(u_{max}^{1-2n}). \quad (32)$$

Eliminate u_c by using

$$L = \frac{R^{n/2}}{u_c^{(n-2)/2}} \int_1^{u_{max}/u_c} dy \sqrt{\frac{(1-x^n)(1-A(n)^2)}{(y^n-x^n)(y^n-x^n-(1-x^n)(1-A(n)^2))}}. \quad (33)$$

The result involves complicated functions of A which can be cast in the following form,

$$E_{F1}(u_{max}) \sim -u_{max}^{1-n} \left(R^{n^2/(n-2)} f_1(A) + u_T^n f_2(A) \right), \quad (34)$$

where $f_{1,2}(A)$ are some functions of A .

Interestingly, the mass dependence of multi-quark potentials has similar form as the mass dependence of mesonic state $\sim m^{1-n}$ in Ref. [20]. This is natural due to the fact that most of the mass of constituent quarks come from the tail part of strings which extend to the large- u region. The mass dependence of the binding potential at the leading order is therefore determined only by the contribution of the hanging strings from the large- u region. As long as the background spacetime of the gravity dual is asymptotically similar to the background considered here in the large- u limit, we would expect the same mass dependence as the form we obtained in this section.

6 Phase diagram

A natural question to ask is whether we have a phase where the exotic multi-quark states are preferred over the normal nuclear matter (namely the gas of N -baryons), vacuum, and the chiral-symmetric quark-gluon plasma phase. To consider a realistic model where these three phases are distinct, we focus our consideration on the Sakai-Sugimoto model with $n = 3$. To calculate the phase diagram involving exotic states, it is necessary to consider the contribution from D8 and $\overline{\text{D8}}$ -branes in the Sakai-Sugimoto model in addition to the contributions from strings and D4-branes. We will assume that the characteristic distance between D8 and $\overline{\text{D8}}$ in x^4 direction is L_0 . The relevant scales of the model therefore depend on both u_T and L_0 .

When there is no radial string pulling the vertex down towards the horizon, it was demonstrated in Ref. [7] by numerical method that the vertex will be pulled all the way up to the position of the flavour branes if the temperature is not very high. Addition of radial strings to the vertex would pull the vertex and the flavour branes towards the horizon. As temperature rises, the radial strings pull the vertex down with stronger force since they are closer to the horizon. It is possible that the vertex then starts to separate from the flavour branes and we might need to consider the configuration where the vertex and flavour branes are separated. However, we can see that the difference between the two configurations should be relatively small (namely, only the force conditions will be slightly different) and we should be able to approximate the situation by considering the configuration where the vertex is not separated from the flavour branes. It is also shown in the Appendix that this configuration satisfies the force condition and thus is allowed. Therefore, it will be assumed that the vertex is always in the flavour branes for the discussion in this section. Moreover, the vertex will be treated as a static configuration and any distortion caused by the strings attached to it will be ignored.

The calculations presented in this section are adapted from Ref. [19] except that we add radial strings hanging from the vertex down to the horizon for the consideration of exotic nuclear phase. We also use position of the D4, u_c , instead of u_0 (where $x'_4(u_0) \rightarrow \infty$) in our

calculation concerning the exotics. This approach allows us to deal with the contribution from radial strings more conveniently. As is shown in Fig. 7, the vacuum phase with broken chiral symmetry corresponds to the configuration where D8 and $\overline{\text{D8}}$ are connected into a curve in the $x_4 - u$ projection. The chiral-symmetric phase of quark-gluon plasma (χ S-QGP) corresponds to the configuration with the parallel D8 and $\overline{\text{D8}}$ stretching from the spacetime boundary down to the horizon. Finally, the nuclear (including exotics) phase corresponds to the configuration where the D4 vertex is located at the D8- $\overline{\text{D8}}$ curve, pulling it down towards the horizon by its “weight” in the background. Each vertex has radial strings attached to it, pulling it further towards the horizon. When there is no radial strings attached, the nuclear phase is of normal N -baryons. The chiral symmetry is also broken in this phase.

Under the above assumptions, the contribution from the strings hanging down from the spacetime boundary to the vertex is negligible. The only contribution of strings is from the radial strings hanging down from the vertex to the horizon. The total action of the configuration is given by

$$S = S_{D8} + S_{D4} + \tilde{S}_{F1}. \quad (35)$$

Generically, the DBI action of the D8-branes is given by

$$S_{D8} = -\mu_8 \int d^9 X e^{-\phi} \text{Tr} \sqrt{-\det(g_{MN} + 2\pi\alpha' F_{MN})} \quad (36)$$

where the field strength of the flavour group $U(N_f)$ is

$$\mathcal{F} = d\mathcal{A} + i\mathcal{A} \wedge \mathcal{A}. \quad (37)$$

The flavour branes provide “global” quantum numbers such as baryon number to the string and subsequently to the strings-brane configuration dual to baryon in the gauge theory side. The diagonal part of the representation matrix of $U(N_f)$ is the $U(1)$ subgroup which induces baryon number to the end of string attached to the flavour branes. Redefine the $U(1)$ part so that

$$\mathcal{A} = \mathcal{A}_{SU(N_f)} + \frac{1}{\sqrt{2N_f}} \hat{\mathcal{A}} \quad (38)$$

with $\hat{\mathcal{A}}$ represents the $U(1)$ piece of the gauge field. The DBI action of the D8-brane coupled to the diagonal gauge field is then given by

$$S_{D8} = \mathcal{N} \int du u^4 \sqrt{f(u)(x'_4(u))^2 + u^{-3}(1 - (\hat{a}'_0(u))^2)} \quad (39)$$

where the constant scales linearly with N_f as

$$\mathcal{N} = \frac{\mu_8 \tau N_f \Omega_4 V_3 R^5}{g_s}, \quad (40)$$

and the rescaled $U(1)$ diagonal field,

$$\hat{a} = \frac{2\pi\alpha'\hat{\mathcal{A}}}{R\sqrt{2N_f}}. \quad (41)$$

The action does not depend on $\hat{a}_0(u)$ explicitly and therefore a constant of motion can be defined as

$$d = \frac{u\hat{a}'_0(u)}{\sqrt{f(u)(x'_4(u))^2 + u^{-3}(1 - (\hat{a}'_0(u))^2)}}. \quad (42)$$

We will see below that the constant d can be interpreted as the baryon number density sourced by the D4-branes once we introduce the Chern-Simon action of the gauge field. Note that d plays the role of the electric displacement field [19]. In the confined phase, the only possible source for d is the D4-brane wrapped on S^4 in the D8-branes. In the deconfined phase, either D4-brane or strings which stretch from the D8-brane down to the horizon can serve as the sources for d . Here, in the study of exotic baryons, we consider the case where *both* D4-brane and strings are present as the sources. This possibility was not investigated in [19].

Similarly, the constant of motion with respect to $x_4(u)$ leads to

$$(x'_4(u))^2 = \frac{1}{u^3 f(u)} \left[\frac{f(u)(u^8 + u^3 d^2)}{f(u_0)(u_0^8 + u_0^3 d^2)} - 1 \right]^{-1} \quad (43)$$

where u_0 is the position when $x'_4(u_0) = \infty$.

Instead of using u_0 as the reference position, the radial position of the D4 on the D8-branes, u_c , can be used to calculate $x'_4(u)$,

$$(x'_4(u))^2 = \frac{1}{u^3 f(u)} \left[\frac{f(u)(u^8 + u^3 d^2)}{F^2} - 1 \right]^{-1} \quad (44)$$

where

$$F = \frac{f(u_c)\sqrt{u_c^8 + u_c^3 d^2}}{\sqrt{f(u_c)(x'_4(u_c))^2 + u_c^{-3}}} x'_4(u_c) \quad (45)$$

$$= \frac{\sqrt{u_c^3 f(u_c)}}{3} \left[1 + \frac{1}{2} \left(\frac{u_T}{u_c} \right)^3 + 3n_s \sqrt{f(u_c)} \right] \sqrt{\frac{9(u_c^5 + d^2)}{1 + \frac{1}{2} \left(\frac{u_T}{u_c} \right)^3 + 3n_s \sqrt{f(u_c)}} - \frac{d^2}{f(u_c)}}. \quad (46)$$

The number of radial strings n_s represents the number of strings hanging down from D4-branes to the horizon in unit of $1/N$. For k , $(N + \bar{k})$ -baryon and j -mesonance, the values of n_s are $1 - k/N$, \bar{k}/N , 1 respectively. Calculation of $x'_4(u_c)$ is performed by minimizing the action with respect to the variation of u_c (see Appendix). For a fixed L_0 , increasing the number of strings n_s results in D4-D8 configuration being pulled down more towards the horizon.

The $U(N_f)$ gauge field \mathcal{A} also generates Chern-Simon term,

$$S_{CS} = \frac{N}{24\pi^2} \int_{M^4 \times R} \omega_5(\mathcal{A}). \quad (47)$$

For $\mathcal{A} = \mathcal{A}_\mu dx^\mu + \mathcal{A}_u du$, the 5-form field strength is given by

$$\omega_5(\mathcal{A}) = \text{Tr} \left(\mathcal{A} \mathcal{F}^2 - \frac{1}{2} \mathcal{A}^3 \mathcal{F} + \frac{1}{10} \mathcal{A}^5 \right). \quad (48)$$

Only the first term contains non-vanishing contribution from the $U(1)$ part which would be identified with the number density of baryon. We will assume a uniform distribution n_4 of the gas of D4-branes in \mathbb{R}^3 at $u = u_c$ in the radial direction. This leads to the relation between the number density of D4-branes, n_4 , and baryon number density d [19],

$$n_4 = \frac{2\pi\alpha' R^2 \mathcal{N}}{\tau V_3 N} d. \quad (49)$$

Phase transition for a system where the number of particles varies is most conveniently described by the grand canonical ensemble. The grand canonical potential of each phase can be defined using the corresponding action of the D8-branes as

$$\Omega(\mu) = \frac{1}{\mathcal{N}} S_{D8}[x_4(u), \hat{a}_0(u)]_{cl}. \quad (50)$$

The baryon chemical potential is given by the $U(1)$ diagonal field at the boundary,

$$\mu = \hat{a}_0(\infty), \quad (51)$$

from which the baryon number density is determined,

$$d = -\frac{\partial \Omega(\mu)}{\partial \mu}. \quad (52)$$

This justifies the association of grand canonical potential with the D8 action. When additional sources of the baryon number are introduced, the free energy, \mathcal{F}_E , from the sources will also contribute to the baryon chemical potential,

$$\mu = \frac{\partial}{\partial d} \frac{1}{\mathcal{N}} \left(\tilde{S}_{D8}[x_4(u), d(u)]_{cl} + S_{source}(d, u_c) \right) \equiv \frac{\partial \mathcal{F}_E}{\partial d}, \quad (53)$$

where the Legendre-transformed action \tilde{S}_{D8} is given by

$$\tilde{S}_{D8} = S_{D8} + \mathcal{N} \int_{u_c}^{\infty} d(u) \hat{a}'_0 du, \quad (54)$$

$$= \mathcal{N} \int_{u_c}^{\infty} du u^4 \sqrt{f(u)(x'_4(u))^2 + u^{-3}} \sqrt{1 + \frac{d^2}{u^5}}. \quad (55)$$

In our case, the additional sources are D4 and radial strings. These relations can also be applied to the vacuum phase (with broken chiral symmetry) where u_c is replaced with u_0 .

Setting $L_0 = 2 \int_{u_i=u_0, u_c}^{\infty} x'_4(u) du = 1$, the expressions for the grand canonical potential and the chemical potential for each phase are given by

vacuum phase, $d = 0$:

$$\Omega_{vac} = \int_{u_0}^{\infty} du \frac{u^{5/2} \sqrt{f(u)}}{\sqrt{f(u) - \frac{u_0^8}{u^8} f(u_0)}}, \quad (56)$$

χ S-QGP phase, $x'_4(u) = 0$:

$$\Omega_{qgp} = \int_{u_T}^{\infty} du \frac{u^5}{\sqrt{u^5 + d^2}}, \quad (57)$$

$$\mu_{qgp} = \int_{u_T}^{\infty} du \frac{d}{\sqrt{u^5 + d^2}}, \quad (58)$$

nuclear (including exotics) phase :

$$\Omega_{nuc} = \int_{u_c}^{\infty} du \left[1 - \frac{F^2}{f(u)(u^8 + u^3 d^2)} \right]^{-1/2} \frac{u^5}{\sqrt{u^5 + d^2}}, \quad (59)$$

$$\mu_{nuc} = \int_{u_c}^{\infty} du \left[1 - \frac{F^2}{f(u)(u^8 + u^3 d^2)} \right]^{-1/2} \frac{d}{\sqrt{u^5 + d^2}} + \frac{1}{3} u_c \sqrt{f(u_c)} + n_s(u_c - u_T). \quad (60)$$

At a fixed temperature T and chemical potential μ , a first order phase transition line between phase 1 and 2 is obtained when $\Omega_1 = \Omega_2, \mu_1 = \mu_2 = \mu$. Transitions between vacuum $\leftrightarrow \chi$ S-QGP and χ S-QGP \leftrightarrow nuclear phases are of this kind. On the other hand, phase transition between nuclear \leftrightarrow vacuum is second order in nature, at least for this case when there is no interaction between each D4. The second order phase transition line occurs when

$$\frac{\partial \mu}{\partial d} = \frac{\partial^2 \mathcal{F}_E}{\partial d^2} \quad (61)$$

has discontinuity at $d = 0$.

In the Sakai-Sugimoto model, there is a phase transition temperature above which gluons become deconfined. However, it does not necessarily imply that everything including quark and antiquark is totally free and chiral symmetry is completely restored above this temperature. When the baryon chemical potential is sufficiently high, baryons can exist even when the temperature is higher than the deconfinement temperature [19]. Only when the temperature increases even further that everything will be completely dissolved and the chiral symmetry is also restored. We also see this behavior in the phase diagram in Figure 8 where we ignore the confined region at low temperature and present only the deconfined part of the phase diagram.

The phase diagram of vacuum with broken chiral symmetry, χ S-QGP and phase of nuclear including exotic multi-quark states is shown in Figure 8. The phase diagram involving

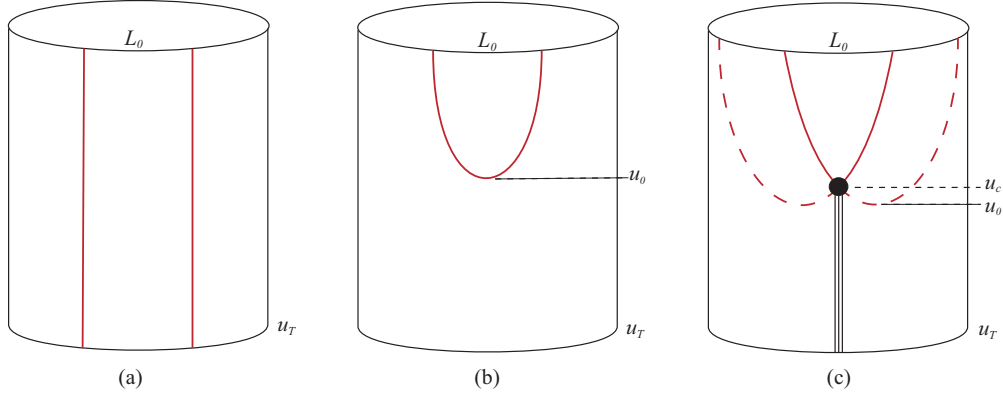


Figure 7: Configurations of χ S-QGP (a), vacuum (b) and exotic nuclear phase (c) in $x^4 - u$ projection.

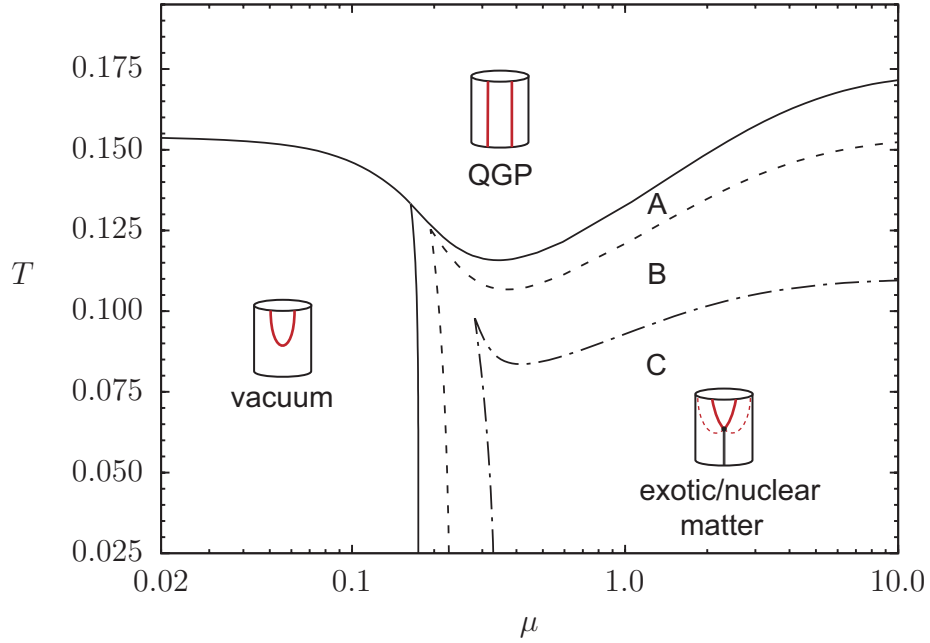


Figure 8: The phase diagram of exotic nuclear matters above the deconfinement temperature. Nuclear phase including exotics is shown as the region on the lower right corner where it is divided into 3 parts for representative purpose. A, B, C represents the region where exotic baryon phase with $n_s = 0$ (N -baryon), 0.1, 0.3 is preferred over vacuum and χ S-QGP respectively.

vacuum and χ S-QGP phases was first obtained in Ref. [18] and the full phase diagram without the exotics was obtained in Ref. [19]. Since the strings pull down the D4-D8 configuration towards the horizon, the configuration with $n_s > 0$ is less stable than the normal N -baryon ($n_s = 0$). This is shown in Fig. 8 where the region of $n_s > 0$ nuclear phase (B, C) is smaller than the region of N -baryon phase (A). They are actually less stable than the N -baryon since the grand canonical potential $\Omega_{n_s>0}(T, \mu) > \Omega_{n_s=0}(T, \mu)$ for $0.5 > n_s > 0$. Above $n_s > 0.3$, the exotic phase becomes unstable to density fluctuations ($\frac{\partial \mu}{\partial d} < 0$) at high temperatures in certain range of d but still remains stable in a region of parameter space. Numerical studies reveal that for approximately $n_s > 0.5$, the multi-quark states become unstable thermodynamically with respect to density fluctuations for most of the temperatures.

Addition of radial strings introduces extra source of the baryonic chemical potential. We can see from Fig. 8 that the value of μ_{onset} for the exotic nuclear phase increases with the value of n_s . Nevertheless, once emerged (i.e. $\mu > \mu_{onset}$), the exotic phases are more stable than the vacuum at any temperature, but less stable than χ S-QGP at sufficiently high temperatures above which chiral symmetry is restored.

7 Discussions

It is desirable to compare the binding energy of each multi-quark state in order to discuss the stability of each configuration as well as their relative abundances in the deconfined phase. At a fixed temperature T , we can compare numerically the binding energies E as functions of the size L of the configuration as is shown in Fig. 2,3. For k -baryon and $(N + \bar{k})$ -baryon, we compare the energy with N -baryon. For j -mesonance, we compare the energy with the energy of j mesons.

From Fig. 2, N -baryon is more energetically favoured than k -baryon and $(N + \bar{k})$ -baryon for any value of k, \bar{k} . Since there are less hanging strings from the spacetime boundary and more radial strings pulled down into the horizon in the case of k -baryon, the vertex is located closer to the horizon and consequently becomes less energetically favoured comparing to the N -baryon. Similarly in the case of $(N + \bar{k})$, even though not as obvious, adding \bar{k} hanging and radial strings to the configuration of N -baryon results in positive energy increase in the binding potential, making this configuration less favoured energetically. An $(N + \bar{k})$ -baryon naturally tends to decay into N -baryon plus \bar{k} free antiquark strings. A k -baryon also has the tendency to fuse with $(N - k)$ quarks to form an N -baryon with lower energy.

The situation of j -mesonance is somewhat similar. Even though j mesons are always energetically preferred over j -mesonance for all values of j , j -mesonance with higher value of j has stronger binding force than the lower ones as is shown in Fig. 3. From the energy viewpoint, j -mesonance will prefer to split into a number of j mesons. It is notable that the screening length of j -mesonance will approach the value of meson, L_{meson}^* , but it will never exceed L_{meson}^* .

For the case of $(N + \bar{k})$ -baryon and j -mesonance, there exist the limits $\bar{k} \rightarrow \infty$ and $j \rightarrow \infty$. The first limit for $(N + \bar{k})$ -baryon leads to the zero-size configuration which saturates the zero-force condition. The second limit for j -mesonance leads to the *mesonic* limit where the

configuration is similar to the system of j mesons as we will see in the following.

From Eqn. (28), since $A(n) \sim (j/N)^{-1}$, $A(n)$ becomes negligible for large j/N . Therefore, we can neglect $A(n)$ and obtain that E_{F1} does not depend on j/N . Using asymptotic expansions, Eqn. (27) becomes

$$\begin{aligned}\mathcal{E} &\simeq \left\{ \int_1^\infty dy \left[\sqrt{\frac{y^n - x^n}{y^n - 1}} - 1 \right] - (1 - x) \right\} \\ &= \left\{ u_T - \frac{\Gamma(\frac{1}{2}) \Gamma(1 - \frac{1}{n})}{\Gamma(\frac{1}{2} - \frac{1}{n})} \frac{C^{2/(n-2)}}{L^{2/(n-2)}} \right\} + \mathcal{O}(x^n),\end{aligned}\tag{62}$$

where

$$C(n) \equiv \frac{R^{n/2} \Gamma(1 - \frac{1}{n}) \Gamma(\frac{1}{2})}{n \Gamma(\frac{3}{2} - \frac{1}{n})}.$$

Now, consider Eqn. (26), we find the screening length L_* (half the distance between quarks at which the binding energy is zero) by setting $E_{tot} = 0$. In the limit of j/N becoming very large, we can obtain L_* from the condition

$$\mathcal{E}(L_*) = 0,\tag{63}$$

leading to

$$L_* \simeq \left[\frac{\Gamma(\frac{1}{2}) \Gamma(1 - \frac{1}{n})}{u_T(n) \Gamma(\frac{1}{2} - \frac{1}{n})} \right]^{(n-2)/2} C(n).\tag{64}$$

Again, the case $n = 3$ and $n = 4$ correspond to the Sakai-Sugimoto and the AdS-Schwarzschild gravity dual model respectively. This expression is exactly the same as the screening length of meson in the deconfined phase from Ref. [20]³. It is no surprise since in the $j \rightarrow \infty$ limit, the hanging strings from the boundary exert force overwhelmingly, therefore the “weight” of the baryon vertex plus the tension of radial strings become negligible. Effectively, the end of hanging string at the vertex will feel zero force down and thus the slope u'_c will be zero. As a result, the strings from the boundary will hang smoothly and appear similar to hanging strings in the case of the mesonic state.

Even in the deconfined phase, we therefore perceive that in addition to free quarks, antiquarks, and gluons, there will also be mesons and multi-quark states. Due to the lower energy, there are more N -baryons than $(N + \bar{k})$ -baryons and k -baryons. The relative populations can be estimated using the Boltzmann factor

$$\exp\left(-\frac{E}{k_B T}\right),\tag{65}$$

determined by the corresponding binding energy E for each state.

A more precise way of considering the deconfined phase is to use the grand canonical potential as the indicator for the stable phase. Following Bergman, Lifschytz, and Lipert [19], we consider three phases of the deconfined soup, a vacuum phase and a nuclear

³Our definition of the screening length is one-half of the definition in Ref. [20].

phase with broken chiral symmetry, and a χ S-QGP. For sufficiently high chemical potential and moderate temperature, the nuclear phase of the multi-quark states is preferred over the vacuum and χ S-QGP phase. Exotic nuclear states such as k -baryon, $(N + \bar{k})$ -baryon, and j -mesonance are characterized by the number of radial strings n_s hanging down from the D4-branes to the horizon. It is found that the multi-quark states with $n_s > 0.5$ are unstable thermodynamically. However, all of these exotic states with $0.5 \geq n_s > 0$ are less stable than the normal N -baryon with $n_s = 0$.

For each value of n_s , there exists a triple point where the grand canonical potentials of the three phases are equivalent. Varying n_s , this triple point will move along the phase transition line between the vacuum and the χ S-QGP as is shown in Fig. 8. The stable region of the nuclear phase shrinks as n_s increases. As $n_s > 0.5$, the nuclear phase becomes thermodynamically unstable with respect to the density fluctuations for most of the parameter space.

8 Conclusion

The gravity dual picture of the deconfined phase suggests that the binding energy or potential between quarks and antiquarks in this phase is nonzero due to the Coulombic piece of the interaction. Since the colorless condition is not required in the deconfined phase, exotic configurations of the multi-quark states are possible. We investigate three classes of these configurations, k -baryon, $(N + \bar{k})$ -baryon, and j -mesonance. It is found that all of these configurations are less energetically favoured than the normal N -baryon as well as being less stable thermodynamically.

The dependence of the screening length on the parameters k, \bar{k}, j is studied and the results are shown in Fig. 4-6. The screening length of k -baryon and j -mesonance are notably increasing with the values of k and j whereas the screening length of $(N + \bar{k})$ -baryon is a decreasing function of \bar{k} . Interestingly, j -mesonance has saturated value of screening length equal to the screening length of meson as $j \rightarrow \infty$.

The dependence on the quark mass of the binding potential at the leading order is derived and found to be $\sim m^{1-n}$ ($n = 3, 4$ for the Sakai-Sugimoto, AdS-Schwarzschild model). The linear quark-mass dependence of the rest energy that we naturally expect is included in the regulator and therefore not present in the binding potential.

In order to consider phase diagram involving exotic nuclear phase, we consider the Sakai-Sugimoto model where the flavour branes D8 and $\overline{D8}$ are introduced. The flavour D8-branes action is identified with the grand canonical potential of the relevant phase. The nuclear phase is considered in the limit when the D4-branes are pulled all the way up to the flavour branes. Exotic multi-quark states with a number of strings stretched down to the horizon, i.e. $n_s > 0$, become less stable than normal N -baryon ($n_s = 0$) since radial strings attached to the D4-branes pull the D4-D8 configuration closer to the horizon. Nevertheless, comparing to the vacuum and the χ S-QGP phase, the nuclear phase of exotic multi-quark states can be more stable in a region of phase diagram with high chemical potential and low temperature as is shown in Fig. 8. In this region, we expect to have a nuclear phase where N -baryons,

k -baryons, and $(N + \bar{k})$ -baryons coexist. For j -mesonance with $n_s = 1$, our consideration of the grand canonical potential suggests that it is thermodynamically unstable to density fluctuations since $\frac{\partial \mu}{\partial d} < 0$. Generically, numerical studies reveal that exotic baryons with $n_s > 0.5$ (namely k -baryon with $k/N < 0.5$, $(N + \bar{k})$ -baryon with $\bar{k}/N > 0.5$ and any j -mesonance) in the deconfined phase are thermodynamically unstable to density fluctuations.

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A Force condition at the D8-branes

There are three forces acting on a D4 locating inside the D8-branes, one from the D8, another from the radial strings pulling down towards horizon and lastly the force from its own “weight” in the background. The equilibrium can be sustained only when these three forces are balanced. As is shown in Ref. [19], variation of the total action with respect to u_c and the constant of motion with respect to $x_4(u)$ lead to

$$x'_4(u_c) = \left(\tilde{L}(u_c) - \frac{\partial S_{source}}{\partial u_c} \right) \bigg/ \frac{\partial \tilde{S}_{D8}}{\partial x'_4} \bigg|_{u_c}, \quad (66)$$

$$= \frac{1}{d} \sqrt{\frac{9u_c^2(1 + \frac{d^2}{u_c^2})}{1 + \frac{1}{2}(\frac{u_T}{u_c})^3 + 3n_s \sqrt{f(u_c)}}} - \frac{d^2 u_c^{-3}}{f(u_c)} \quad (67)$$

where the Legendre transformed action is

$$\tilde{S}_{D8} = \int_{u_c}^{\infty} \tilde{L}(x'_4(u), d) du, \quad (68)$$

$$= \mathcal{N} \int_{u_c}^{\infty} du u^4 \sqrt{f(u)(x'_4(u))^2 + u^{-3}} \sqrt{1 + \frac{d^2}{u^5}}, \quad (69)$$

and the source term is given by

$$S_{source} = \mathcal{N} d \left[\frac{1}{3} u_c \sqrt{f(u_c)} + n_s (u_c - u_T) \right]. \quad (70)$$

There are two contributions from the D-branes and strings as the sources for the baryon chemical potential. Additional strings increase the baryonic chemical potential of the exotic

multi-quark states. Since the number of total charge on each D4 is N which is absorbed into \mathcal{N} , the number of radial strings stretched down to the horizon, n_s , is thus given in unit of $1/N$.

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